CHAPTER 7

Systems of Particles and Rotational Motion

7.2 Centre of Mass

1. Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of
(a) 33 cm (b) 50 cm
(c) 67 cm (d) 80 cm *(NEET 2020)*

2. Three masses are placed on the x-axis : 300 g at origin, 500 g at x = 40 cm and 400 g at x = 70 cm. The distance of the centre of mass from the origin is
(a) 40 cm (b) 45 cm
(c) 50 cm (d) 30 cm *(Mains 2012)*

3. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$, respectively. The centre of mass of this system has a position vector
(a) $-2\hat{i}-\hat{j}+\hat{k}$ (b) $2\hat{i}-\hat{j}-2\hat{k}$
(c) $-\hat{i}+\hat{j}+\hat{k}$ (d) $-2\hat{i}+2\hat{k}$ *(2009)*

4. Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass of the particles through a distance d, by what distance would be particle of mass m_2 move so as to keep the centre of mass of the particles at the original position ?
(a) $\frac{m_1}{m_1+m_2}d$ (b) $\frac{m_1}{m_2}d$
(c) d (d) $\frac{m_2}{m_1}d$ *(2004)*

5. Three identical metal balls, each of radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed when centres of three balls are joined. The centre of the mass of the system is located at
(a) line joining centres of any two balls
(b) centre of one of the balls
(c) horizontal surface
(d) point of intersection of the medians. *(1999)*

6. The centre of mass of system of particles does not depend on
(a) position of the particles
(b) relative distances between the particles
(c) masses of the particles
(d) forces acting on the particle. *(1997)*

7.3 Motion of Centre of Mass

7. Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3.0 m and weighs 100 kg. The 55 kg man walks up to the 65 kg man and sits with him. If the boat is in still water the center of mass of the system shifts by
(a) 3.0 m (b) 2.3 m (c) zero (d) 0.75 m *(2012)*

8. Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and $2v$ at any instant, then the speed of centre of mass of the system will be
(a) $2v$ (b) zero (c) $1.5v$ (d) v *(2010)*

9. A man of 50 kg mass is standing in a gravity free space at a height of 10 m above the floor. He throws a stone of 0.5 kg mass downwards with a speed 2 m/s. When the stone reaches the floor, the distance of the man above the floor will be
(a) 9.9 m (b) 10.1 m
(c) 10 m (d) 20 m *(2010)*

7.5 Vector Product of Two Vectors

10. Vectors, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that $\vec{A}\cdot\vec{B}=0$ and $\vec{A}\cdot\vec{C}=0$. Then the vector parallel to $\vec{A}$ is
(a) $\vec{A}\times\vec{B}$ (b) $\vec{B}+\vec{C}$
(c) $\vec{B}\times\vec{C}$ (d) $\vec{B}$ and $\vec{C}$
(Karnataka NEET 2013)

11. $\vec{A}$ and $\vec{B}$ are two vectors and θ is the angle between them, if $|\vec{A}\times\vec{B}| = \sqrt{3}(\vec{A}\cdot\vec{B})$, the value of θ is
(a) 45° (b) 30° (c) 90° (d) 60° *(2007)*

12. If the angle between the vectors $\vec{A}$ and $\vec{B}$ is θ, the value of the product $(\vec{B}\times\vec{A})\cdot\vec{A}$ is equal to
(a) $BA^2\sin\theta$ (b) $BA^2\cos\theta$
(c) $BA^2\sin\theta\cos\theta$ (d) zero. *(2005, 1989)*

13. If $|\vec{A}\times\vec{B}|=\sqrt{3}\vec{A}\cdot\vec{B}$ then the value of $|\vec{A}+\vec{B}|$ is
(a) $(A^2+B^2+AB)^{1/2}$ (b) $\left(A^2+B^2+\frac{AB}{\sqrt{3}}\right)^{1/2}$
(c) $A+B$ (d) $\left(A^2+B^2+\sqrt{3}AB\right)^{1/2}$ *(2004)*

14. The resultant of $\vec{A}\times 0$ will be equal to
(a) zero (b) A
(c) zero vector (d) unit vector. *(1992)*

7.6 Angular Velocity and its Relation with Linear Velocity

15. What is the value of linear velocity, if $\vec{r}=3\hat{i}-4\hat{j}+\hat{k}$ and $\vec{\omega}=5\hat{i}-6\hat{j}+6\hat{k}$?
(a) $4\hat{i}-13\hat{j}+6\hat{k}$ (b) $18\hat{i}+13\hat{j}-2\hat{k}$
(c) $6\hat{i}+2\hat{j}-3\hat{k}$ (d) $6\hat{i}-2\hat{j}+8\hat{k}$ *(1999)*

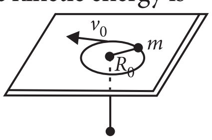
7.7 Torque and Angular Momentum

16. Find the torque about the origin when a force of $3\hat{j}$ N acts on a particle whose position vector is $2\hat{k}$ m.
(a) $6\hat{i}$ N m (b) $6\hat{j}$ N m
(c) $-6\hat{i}$ N m (d) $6\hat{k}$ N m *(NEET 2020)*

17. The moment of the force, $\vec{F}=4\hat{i}+5\hat{j}-6\hat{k}$ at (2, 0, –3), about the point (2, –2, –2), is given by
(a) $-8\hat{i}-4\hat{j}-7\hat{k}$ (b) $-4\hat{i}-\hat{j}-8\hat{k}$
(c) $-7\hat{i}-8\hat{j}-4\hat{k}$ (d) $-7\hat{i}-4\hat{j}-8\hat{k}$
(NEET 2018)

18. A force $\vec{F}=\alpha\hat{i}+3\hat{j}+6\hat{k}$ is acting at a point $\vec{r}=2\hat{i}-6\hat{j}-12\hat{k}$. The value of α for which angular momentum about origin is conserved is
(a) zero (b) 1 (c) –1 (d) 2 *(2015)*

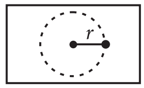
19. A mass m moves in a circle on a smooth horizontal plane with velocity v_0 at a radius R_0. The mass is attached to a string which passes through a smooth hole in the plane as shown. The tension in the string is increased gradually and finally m moves in a circle of radius $\frac{R_0}{2}$. The final value of the kinetic energy is



(a) $2mv_0^2$
(b) $\frac{1}{2}mv_0^2$
(c) mv_0^2 (d) $\frac{1}{4}mv_0^2$ *(2015 Cancelled)*

20. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along
(a) a line perpendicular to the plane of rotation
(b) the line making an angle of 45° to the plane of rotation
(c) the radius
(d) the tangent to the orbit. *(2012)*

21. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will



(a) decrease by a factor of 2
(b) remain constant
(c) increase by a factor of 2
(d) increase by a factor of 4 *(Mains 2011)*

22. If $\vec{F}$ is the force acting on a particle having position vector $\vec{r}$ and $\vec{\tau}$ be the torque of this force about the origin, then
(a) $\vec{r}\cdot\vec{\tau}>0$ and $\vec{F}\cdot\vec{\tau}<0$ (b) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{\tau}=0$
(c) $\vec{r}\cdot\vec{\tau}=0$ and $\vec{F}\cdot\vec{\tau}\neq 0$ (d) $\vec{r}\cdot\vec{\tau}\neq 0$ and $\vec{F}\cdot\vec{\tau}=0$
(2009)

23. A particle of mass m moves in the XY plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then
(a) $L_A=L_B$
(b) the relationship between L_A and L_B depends upon the slope of the line AB
(c) $L_A<L_B$ (d) $L_A>L_B$ *(2007)*

24. Find the torque of a force $\vec{F}=-3\hat{i}+\hat{j}+5\hat{k}$ acting at the point $\vec{r}=7\hat{i}+3\hat{j}+\hat{k}$.
(a) $-21\hat{i}+4\hat{j}+4\hat{k}$ (b) $-14\hat{i}+34\hat{j}-16\hat{k}$
(c) $14\hat{i}-38\hat{j}+16\hat{k}$ (d) $4\hat{i}+4\hat{j}+6\hat{k}$ *(1997)*

25. What is the torque of the force $\vec{F}=2\hat{i}-3\hat{j}+4\hat{k}$ N acting at the point $\vec{r}=3\hat{i}+2\hat{j}+3\hat{k}$ m about origin?
(a) $-6\hat{i}+6\hat{j}-12\hat{k}$ (b) $-17\hat{i}+6\hat{j}+13\hat{k}$
(c) $6\hat{i}-6\hat{j}+12\hat{k}$ (d) $17\hat{i}-6\hat{j}-13\hat{k}$ *(1995)*

26. A particle of mass m = 5 is moving with a uniform speed $v=3\sqrt{2}$ in the XOY plane along the line $y=x+4$. The magnitude of the angular momentum of the particle about the origin is
(a) 60 units (b) $40\sqrt{2}$ units
(c) zero (d) 7.5 units *(1991)*

7.8 Equilibrium of a Rigid Body

27. Which of the following statements are correct?
(1) Centre of mass of a body always coincides with the centre of gravity of the body.
(2) Centre of mass of a body is the point at which the total gravitational torque on the body is zero.
(3) A couple on a body produces both translational and rotational motion in a body.
(4) Mechanical advantage greater than one means that small effort can be used to lift a large load.

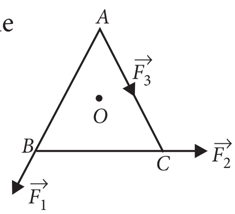
(a) (1) and (2) (b) (2) and (3)
(c) (3) and (4) (d) (2) and (4) *(NEET 2017)*

28. A rod of weight W is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is

(a) $\frac{W(d-x)}{x}$ (b) $\frac{W(d-x)}{d}$

(c) $\frac{Wx}{d}$ (d) $\frac{Wd}{x}$ *(2015 Cancelled)*

29. ABC is an equilateral triangle with O as its centre. $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ represent three forces acting along the sides AB, BC and AC respectively. If the total torque about O is zero then the magnitude of $\vec{F}_3$ is



(a) $F_1 + F_2$ (b) $F_1 - F_2$

(c) $\frac{F_1 + F_2}{2}$ (d) $2(F_1 + F_2)$ *(2012, 1998)*

30. (1) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts.
(2) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius.
(3) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.
(4) The radius of gyration of any body rotating about an axis is the length of the perpendicular drawn from the C.G. of the body to the axis.

Which one of the following pairs of statements is correct?

(a) (4) and (1) (b) (1) and (2)
(c) (2) and (3) (d) (3) and (4) *(Mains 2010)*

31. A rod of length 3 m and its mass per unit length is directly proportional to distance x from one of its end then its centre of gravity from that end will be at

(a) 1.5 m (b) 2 m
(c) 2.5 m (d) 3.0 m *(2002)*

32. 250 N force is required to raise 75 kg mass from a pulley. If rope is pulled 12 m then the load is lifted to 3 m, the efficiency of pulley system will be

(a) 25% (b) 33.3%
(c) 75% (d) 90%. *(2001)*

33. A couple produces
(a) linear and rotational motion
(b) no motion
(c) purely linear motion
(d) purely rotational motion. *(1997)*

7.9 Moment of Inertia

34. A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation $(E_{sphere}/E_{cylinder})$ will be

(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1 *(NEET-II 2016)*

35. A light rod of length l has two masses m_1 and m_2 attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is

(a) $\frac{m_1 m_2}{m_1 + m_2} l^2$ (b) $\frac{m_1 + m_2}{m_1 m_2} l^2$

(c) $(m_1 + m_2)l^2$ (d) $\sqrt{m_1 m_2}\, l^2$ *(NEET-II 2016)*

36. From a circular disc of radius R and mass $9M$, a small disc of mass M and radius $R/3$ is removed concentrically. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through its centre is

(a) $\frac{40}{9} MR^2$ (b) MR^2

(c) $4MR^2$ (d) $\frac{4}{9} MR^2$ *(Mains 2010)*

37. The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is

(a) $\sqrt{2}:1$ (b) $\sqrt{2}:\sqrt{3}$

(c) $\sqrt{3}:\sqrt{2}$ (d) $1:\sqrt{2}$ *(2008)*

38. Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular velocity will be in the ratio

(a) 2 : 1 (b) 1 : 2
(c) $\sqrt{2}$: 1 (d) 1 : $\sqrt{2}$ *(2005)*

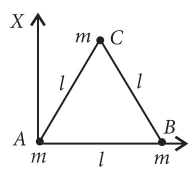
39. Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram-cm^2 units will be

(a) $\frac{3}{4} ml^2$

(b) $2ml^2$

(c) $\frac{5}{4} ml^2$

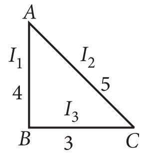
(d) $\frac{3}{2} ml^2$ *(2004)*



40. A circular disc is to be made by using iron and aluminium so that it acquires maximum moment of inertia about geometrical axis. It is possible with

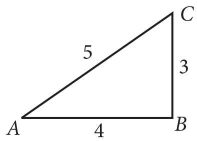
(a) aluminium at interior and iron surrounding it
(b) iron at interior and aluminium surrounding it
(c) using iron and aluminium layers in alternate order
(d) sheet of iron is used at both external surface and aluminium sheet as internal layers. *(2002)*

41. For the adjoining diagram, the correct relation between I_1, I_2, and I_3 is, (I-moment of inertia)



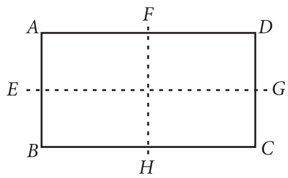
(a) $I_1 > I_2$
(b) $I_2 > I_1$
(c) $I_3 > I_1$
(d) $I_3 > I_2$ *(2000)*

42. The ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA respectively. Which one of the following relations is correct?



(a) $I_{AB} + I_{BC} = I_{CA}$ (b) I_{CA} is maximum
(c) $I_{AB} > I_{BC}$ (d) $I_{BC} > I_{AB}$ *(1995)*

43. In a rectangle $ABCD$ ($BC = 2AB$). The moment of inertia is minimum along axis through



(a) BC
(b) BD
(c) HF
(d) EG *(1993)*

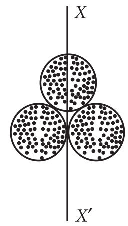
44. A fly wheel rotating about fixed axis has a kinetic energy of 360 joule when its angular speed is 30 radian/sec. The moment of inertia of the wheel about the axis of rotation is
(a) 0.6 kg m^2 (b) 0.15 kg m^2
(c) 0.8 kg m^2 (d) 0.75 kg m^2 *(1990)*

45. A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is
(a) $\frac{1}{2}mr^2\omega^2$ (b) $mr\omega^2$
(c) $mr^2\omega^2$ (d) $\frac{1}{2}mr\omega^2$ *(1988)*

7.10 Theorems of Perpendicular and Parallel Axes

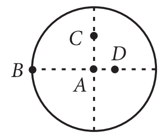
46. From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre?
(a) 11 $MR^2/32$ (b) 9 $MR^2/32$
(c) 15 $MR^2/32$ (d) 13 $MR^2/32$ *(NEET-I 2016)*

47. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching the two shells and passing through the diameter of the third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is



(a) $\frac{16}{5}mr^2$
(b) $4mr^2$
(c) $\frac{11}{5}mr^2$
(d) $3mr^2$ *(2015 Cancelled)*

48. The moment of inertia of a uniform circular disc is maximum about an axis perpendicular to the disc and passing through



(a) B
(b) C
(c) D
(d) A *(Mains 2012)*

49. The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its midpoint and perpendicular to its length is I_0. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is
(a) $I_0 + ML^2/2$ (b) $I_0 + ML^2/4$
(c) $I_0 + 2ML^2$ (d) $I_0 + ML^2$ *(2011)*

50. Four identical thin rods each of mass M and length l, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is
(a) $\frac{2}{3}Ml^2$ (b) $\frac{13}{3}Ml^2$
(c) $\frac{1}{3}Ml^2$ (d) $\frac{4}{3}Ml^2$ *(2009)*

51. A thin rod of length L and mass M is bent at its midpoint into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is
(a) $\frac{ML^2}{6}$ (b) $\frac{\sqrt{2}ML^2}{24}$ (c) $\frac{ML^2}{24}$ (d) $\frac{ML^2}{12}$ *(2008)*

52. The moment of inertia of a uniform circular disc of radius R and mass M about an axis touching the disc at its diameter and normal to the disc
(a) $\frac{1}{2}MR^2$ (b) MR^2
(c) $\frac{2}{5}MR^2$ (d) $\frac{3}{2}MR^2$ *(2006)*

53. The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius and mass about a tangential axis in the plane of the ring is
(a) 2 : 3 (b) 2 : 1
(c) $\sqrt{5}:\sqrt{6}$ (d) $1:\sqrt{2}$ *(2004)*

54. The moment of inertia of a disc of mass M and radius R about an axis, which is tangential to the circumference of the disc and parallel to its diameter is
(a) $\frac{5}{4}MR^2$ (b) $\frac{2}{3}MR^2$
(c) $\frac{3}{2}MR^2$ (d) $\frac{4}{5}MR^2$ *(1999)*

55. Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be
(a) $5I$ (b) $3I$ (c) $6I$ (d) $4I$ *(1990)*

7.11 Kinematics of Rotational Motion about a Fixed Axis

56. A wheel has angular acceleration of 3.0 rad/sec^2 and an initial angular speed of 2.00 rad/sec. In a time of 2 sec it has rotated through an angle (in radians) of
(a) 10 (b) 12 (c) 4 (d) 6 *(2007)*

7.12 Dynamics of Rotational Motion about a Fixed Axis

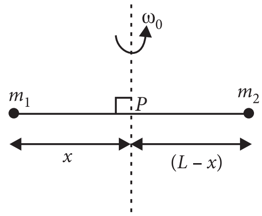
57. A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop it after 2π revolutions is
(a) 2×10^{6} N m (b) 2×10^{-6} N m
(c) 2×10^{-3} N m (d) 12×10^{-4} N m
(NEET 2019)

58. Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation
(a) $W_C > W_B > W_A$ (b) $W_A > W_B > W_C$
(c) $W_B > W_A > W_C$ (d) $W_A > W_C > W_B$
(NEET 2018)

59. A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder if the rope is pulled with a force of 30 N?
(a) 0.25 rad s^{-2} (b) 25 rad s^{-2}
(c) 5 m s^{-2} (d) 25 m s^{-2} *(NEET 2017)*

60. A uniform circular disc of radius 50 cm at rest is free to turn about an axis which is perpendicular to its plane and passes through its centre. It is subjected to a torque which produces a constant angular acceleration of 2.0 rad s^{-2}. Its net acceleration in m s^{-2} at the end of 2.0 s is approximately
(a) 6.0 (b) 3.0 (c) 8.0 (d) 7.0
(NEET-I 2016)

61. Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L, and negligible mass. The rod is to be set rotating about an axis perpendicular to it.

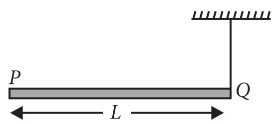


The position of point P on this rod through which the axis should pass so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by
(a) $x = \frac{m_2}{m_1}L$ (b) $x = \frac{m_2 L}{m_1 + m_2}$
(c) $x = \frac{m_1 L}{m_1 + m_2}$ (d) $x = \frac{m_1}{m_2}L$ *(2015)*

62. An automobile moves on a road with a speed of 54 km h^{-1}. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is 3 kg m^2. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is
(a) 10.86 kg m^2 s^{-2} (b) 2.86 kg m^2 s^{-2}
(c) 6.66 kg m^2 s^{-2} (d) 8.58 kg m^2 s^{-2} *(2015)*

63. A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 revolutions s^{-2} is
(a) 25 N (b) 50 N
(c) 78.5 N (d) 157 N *(2014)*

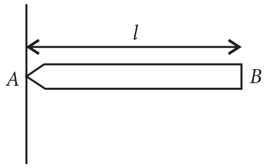
64. A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is



(a) $\frac{2g}{L}$ (b) $\frac{2g}{2L}$ (c) $\frac{3g}{2L}$ (d) $\frac{g}{L}$
(NEET 2013)

65. The instantaneous angular position of a point on a rotating wheel is given by the equation $\theta(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at
(a) $t = 1$ s (b) $t = 0.5$ s
(c) $t = 0.25$ s (d) $t = 2$ s *(2011)*

66. A uniform rod AB of length l and mass m is free to rotate about point A. The rod is released from rest in the horizontal position. Given that the moment of inertia of the rod about A is $ml^2/3$, the initial angular acceleration of the rod will be



(a) $\frac{mgl}{2}$

(b) $\frac{3}{2}gl$

(c) $\frac{3g}{2l}$ (d) $\frac{2g}{3l}$ *(2007, 2006)*

67. A wheel having moment of inertia 2 kg m^2 about its vertical axis, rotates at the rate of 60 rpm about this axis. The torque which can stop the wheel's rotation in one minute would be

(a) $\frac{2\pi}{15}$ N m (b) $\frac{\pi}{12}$ N m

(c) $\frac{\pi}{15}$ N m (d) $\frac{\pi}{18}$ N m *(2004)*

68. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 joule, an angular acceleration of 25 radian/sec^2 must be applied about that axis for a duration of

(a) 4 s (b) 2 s (c) 8 s (d) 10 s *(1990)*

7.13 Angular Momentum in case of Rotation about a Fixed Axis

69. A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere?

(a) Angular velocity. (b) Moment of inertia.
(c) Rotational kinetic energy.
(d) Angular momentum. *(NEET 2018)*

70. Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc has angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is

(a) $\frac{1}{4}I(\omega_1 - \omega_2)^2$ (b) $I(\omega_1 - \omega_2)^2$

(c) $\frac{1}{8}I(\omega_1 - \omega_2)^2$ (d) $\frac{1}{2}I(\omega_1 + \omega_2)^2$

(NEET 2017)

71. Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then

(a) $L_A = \frac{L_B}{2}$ (b) $L_A = 2L_B$

(c) $L_B > L_A$ (d) $L_A > L_B$ *(NEET-II 2016)*

72. Two discs are rotating about their axes, normal to the discs and passing through the centres of the discs. Disc D_1 has 2 kg mass and 0.2 m radius and initial angular velocity of 50 rad s^{-1}. Disc D_2 has 4 kg mass, 0.1 m radius and initial angular velocity of 200 rad s^{-1}. The two discs are brought in contact face to face, with their axes of rotation coincident. The final angular velocity (in rad s^{-1}) of the system is

(a) 60 (b) 100 (c) 120 (d) 40

(Karnataka NEET 2013)

73. A circular platform is mounted on a frictionless vertical axle. Its radius R = 2 m and its moment of inertia about the axle is 200 kg m^2. It is initially at rest. A 50 kg man stands on the edge of the platform and begins to walk along the edge at the speed of 1 m s^{-1} relative to the ground. Time taken by the man to complete one revolution is

(a) π s (b) $3\pi/2$ s
(c) 2π s (d) $\pi/2$ s *(Mains 2012)*

74. A circular disk of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed ω_i. Another disk of moment of inertia I_b is dropped coaxially onto the rotating disk. Initially the second disk has zero angular speed. Eventually both the disks rotate with a constant angular speed ω_f. The energy lost by the initially rotating disc to friction is

(a) $\frac{1}{2}\frac{I_b^2}{(I_t + I_b)}\omega_i^2$ (b) $\frac{1}{2}\frac{I_t^2}{(I_t + I_b)}\omega_i^2$

(c) $\frac{I_b - I_t}{(I_t + I_b)}\omega_i^2$ (d) $\frac{1}{2}\frac{I_b I_t}{(I_t + I_b)}\omega_i^2$ *(2010)*

75. A thin circular ring of mass M and radius r is rotating about its axis with constant angular velocity ω. Two objects each of mass m are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with angular velocity given by

(a) $\frac{(M+2m)\omega}{2m}$ (b) $\frac{2M\omega}{M+2m}$

(c) $\frac{(M+2m)\omega}{M}$ (d) $\frac{M\omega}{M+2m}$

(Mains 2010, 1998)

76. A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

(a) $\frac{I_2\omega}{I_1 + I_2}$ (b) ω

(c) $\frac{I_1\omega}{I_1 + I_2}$ (d) $\frac{(I_1 + I_2)\omega}{I_1}$ *(2004)*

77. A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be

(a) $\frac{M\omega}{4m}$ (b) $\frac{M\omega}{M+4m}$

(c) $\frac{(M+4m)\omega}{M}$ (d) $\frac{(M-4m)\omega}{M+4m}$ *(2003)*

78. A disc is rotating with angular speed ω. If a child sits on it, what is conserved?

(a) linear momentum. (b) angular momentum.
(c) kinetic energy. (d) potential energy. *(2002)*

7.14 Rolling Motion

79. A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it ?

(a) 1 J (b) 3 J (c) 30 kJ (d) 2 J *(NEET 2019)*

80. A solid cylinder of mass 2 kg and radius 50 cm rolls up an inclined plane of angle inclination 30°. The centre of mass of cylinder has speed of 4 m/s. The distance travelled by the cylinder on the incline surface will be (Take $g = 10\ m/s^2$)

(a) 2.2 m (b) 1.6 m (c) 1.2 m (d) 2.4 m *(Odisha NEET 2019)*

81. A solid sphere is in rolling motion. In rolling motion a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is

(a) 7 : 10 (b) 5 : 7 (c) 10 : 7 (d) 2 : 5 *(NEET 2018, 1991)*

82. A disc and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first?

(a) Both reach at the same time
(b) Depends on their masses
(c) Disc (d) Sphere *(NEET-I 2016)*

83. The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is

(a) 5 : 7 (b) 2 : 3 (c) 2 : 5 (d) 7 : 5 *(2014)*

84. A small object of uniform density rolls up a curved surface with an initial velocity 'v'. It reaches upto a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

(a) hollow sphere (b) disc
(c) ring (d) solid sphere. *(NEET 2013)*

85. A solid cylinder of mass 3 kg is rolling on a horizontal surface with velocity 4 m s^{-1}. It collides with a horizontal spring of force constant 200 N m^{-1}. The maximum compression produced in the spring will be

(a) 0.5 m (b) 0.6 m (c) 0.7 m (d) 0.2 m *(2012)*

86. A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping. Which one will reach the bottom first?

(a) Both together only when angle of inclination of plane is 45°.
(b) Both together.
(c) Hollow cylinder.
(d) Solid cylinder. *(Mains 2010)*

87. A drum of radius R and mass M, rolls down without slipping along an inclined plane of angle θ. The frictional force

(a) dissipates energy as heat
(b) decreases the rotational motion
(c) decreases the rotational and translational motion
(d) converts translational energy to rotational energy. *(2005)*

88. A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is K. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be

(a) $\frac{K^2+R^2}{R^2}$ (b) $\frac{K^2}{R^2}$

(c) $\frac{K^2}{K^2+R^2}$ (d) $\frac{R^2}{K^2+R^2}$ *(2003)*

89. A solid cylinder of mass M and radius R rolls without slipping down an inclined plane of length L and height h. What is the speed of its centre of mass when the cylinder reaches its bottom ?

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{3}{4}gh}$

(c) $\sqrt{\frac{4}{3}gh}$ (d) $\sqrt{4gh}$ *(2003, 1989)*

90. Consider a contact point P of a wheel on ground which rolls on ground without slipping. Then value of displacement of point P when wheel completes half of rotation (If radius of wheel is 1 m)

(a) 2 m (b) $\sqrt{\pi^2+4}$ m

(c) π m (d) $\sqrt{\pi^2+2}$ m *(2002)*

91. A solid sphere of radius R is placed on a smooth horizontal surface. A horizontal force F is applied at height h from the lowest point. For the maximum acceleration of centre of mass, which is correct?

(a) $h = R$ (b) $h = 2R$
(c) $h = 0$
(d) no relation between h and R. *(2002)*

92. A disc is rolling, the velocity of its centre of mass is $v_{cm.}$ Which one will be correct?
(a) The velocity of highest point is $2\ v_{cm}$ and at point of contact is zero.
(b) The velocity of highest point is v_{cm} and at point of contact is v_{cm}.
(c) The velocity of highest point is $2v_{cm}$ and point of contact is v_{cm}.
(d) The velocity of highest point is $2v_{cm}$ and point of contact is $2v_{cm}$. *(2001)*

93. A solid spherical ball rolls on a table. Ratio of its rotational kinetic energy to total kinetic energy is
(a) 1/2 (b) 1/6 (c) 7/10 (d) 2/7 *(1994)*

94. A solid sphere, disc and solid cylinder all of the same mass and radius are allowed to roll down (from rest) on the inclined plane, then
(a) solid sphere reaches the bottom first
(b) solid sphere reaches the bottom last
(c) disc will reach the bottom first
(d) all reach the bottom at the same time. *(1993)*

95. The speed of a homogenous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is
(a) $\sqrt{\frac{10}{7}gh}$ (b) $\sqrt{gh}$
(c) $\sqrt{\frac{6}{5}gh}$ (d) $\sqrt{\frac{4}{3}gh}$ *(1992)*

96. A solid homogenous sphere of mass M and radius is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere
(a) total kinetic energy is conserved
(b) the angular momentum of the sphere about the point of contact with the plane is conserved
(c) only the rotational kinetic energy about the centre of mass is conserved
(d) angular momentum about the centre of mass is conserved. *(1988)*

ANSWER KEY

1. (c)	2. (a)	3. (a)	4. (b)	5. (d)	6. (d)	7. (c)	8. (b)	9. (b)	10. (c)
11. (d)	12. (d)	13. (a)	14. (c)	15. (b)	16. (c)	17. (d)	18. (c)	19. (a)	20. (a)
21. (d)	22. (b)	23. (a)	24. (c)	25. (d)	26. (a)	27. (*)	28. (b)	29. (a)	30. (a)
31. (b)	32. (c)	33. (d)	34. (b)	35. (a)	36. (a)	37. (d)	38. (c)	39. (c)	40. (a)
41. (b)	42. (d)	43. (d)	44. (c)	45. (a)	46. (d)	47. (b)	48. (a)	49. (b)	50. (d)
51. (d)	52. (d)	53. (c)	54. (a)	55. (c)	56. (a)	57. (b)	58. (a)	59. (b)	60. (c)
61. (b)	62. (c)	63. (d)	64. (c)	65. (a)	66. (c)	67. (c)	68. (b)	69. (d)	70. (a)
71. (c)	72. (b)	73. (c)	74. (d)	75. (d)	76. (c)	77. (b)	78. (b)	79. (b)	80. (d)
81. (b)	82. (d)	83. (a)	84. (b)	85. (b)	86. (d)	87. (d)	88. (c)	89. (c)	90. (b)
91. (d)	92. (a)	93. (d)	94. (a)	95. (a)	96. (b)				

Hints & Explanations

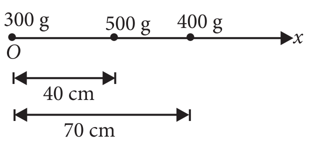
1. (c) : Given : $m_1 = 5$ kg, $m_2 = 10$ kg and $L = 1$ m

Here centre of mass, $X_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$$= \frac{5\times 0 + 10\times 1}{15} = \frac{10}{15} = \frac{2}{3}$$

$= 0.666$ m $\simeq 0.67$ m $= 67$ cm

2. (a) :



The distance of the centre of mass of the system of three masses from the origin O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{300\times 0 + 500\times 40 + 400\times 70}{300 + 500 + 400}$$

$$= \frac{500\times 40 + 400\times 70}{1200} = \frac{400\,[50+70]}{1200}$$

$$= \frac{50+70}{3} = \frac{120}{3} = 40 \text{ cm}$$

3. (a) : $\vec{r}_1 = \hat{i} + 2\hat{j} + \hat{k}$ for $M_1 = 1$ kg

$\vec{r}_2 = -3\hat{i} - 2\hat{j} + \hat{k}$ for $M_2 = 3$ kg

$$r_{C.M.} = \frac{\sum m_i r_i}{\sum m_i}$$

$$\Rightarrow \; r_{C.M.} = \frac{(1\hat{i}+2\hat{j}+1\hat{k})\times 1+(-3\hat{i}-2\hat{j}+\hat{k})\times 3}{4}$$

$$\Rightarrow \; r_{C.M.} = \frac{-8\hat{i}-4\hat{j}+4\hat{k}}{4} = -2\hat{i}-\hat{j}+\hat{k}$$

4. **(b) :** $\text{C.M.} = \dfrac{m_1x_1+m_2x_2}{m_1+m_2}$ ...(i)

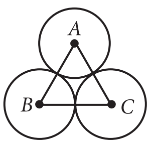
After changing position of m_1 and to keep the position of C.M. same

$$\text{C.M.} = \frac{m_1(x_1-d)+m_2(x_2+d_2)}{m_1+m_2}$$

$$0 = \frac{-m_1d+m_2d_2}{m_1+m_2}$$ [Substituting value of C.M. from (i)]

$$\Rightarrow d_2 = \frac{m_1}{m_2}d$$

5. **(d) :** Centre of mass of each ball lies on the centre.
$\Rightarrow$ centre of mass of combined body will be at the centroid of equilateral triangle.



6. **(d) :** The resultant of all forces, on any system of particles, is zero. Therefore their centre of mass does not depend upon the forces acting on the particles.

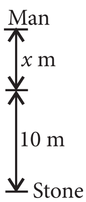
7. **(c) :** As no external force acts on the system, therefore centre of mass will not shift.

8. **(b) :** As no external force is acting on the system, the centre of mass must be at rest *i.e.* $v_{CM} = 0$.

9. **(b) :** Since the man is in gravity free space, force on man + stone system is zero.
Therefore centre of mass of the system remains at rest. Let the man goes x m above when the stone reaches the floor, then

$$M_{man} \times x = M_{stone} \times 10$$

$$x = \frac{0.5}{50}\times 10 \quad \text{or} \quad x = 0.1 \text{ m}$$



Therefore final height of man above floor = 10 + x
= 10 + 0.1 = 10.1 m

10. **(c) :** Vector triple product of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ is

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B}-(\vec{A}\cdot\vec{B})\vec{C}$$

Given : $\vec{A}\cdot\vec{B}=0$, $\vec{A}\cdot\vec{C}=0$

$\therefore \quad \vec{A}\times(\vec{B}\times\vec{C})=0$

Thus the vector $\vec{A}$ is parallel to vector $\vec{B}\times\vec{C}$.

11. **(d) :** $|\vec{A}\times\vec{B}| = \sqrt{3}(\vec{A}\cdot\vec{B})$

$\therefore \quad AB\sin\theta = \sqrt{3}AB\cos\theta$

or, $\tan\theta = \sqrt{3}$ or, $\theta = \tan^{-1}\sqrt{3} = 60°$

12. **(d) :** Let $A\times B = C$

The cross product of $\vec{B}$ and $\vec{A}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$ *i.e.* perpendicular to $\vec{A}$.

Therefore product of $(\vec{B}\times\vec{A})\cdot\vec{A} = 0$

13. **(a) :** $|\vec{A}\times\vec{B}| = \sqrt{3}\vec{A}\cdot\vec{B}$

$|\vec{A}||\vec{B}|\sin\theta = \sqrt{3}|\vec{A}||\vec{B}|\cos\theta$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

$|\vec{A}+\vec{B}| = \sqrt{|\vec{A}|^2+|\vec{B}|^2+2|\vec{A}||\vec{B}|\cos\theta}$

$= (A^2+B^2+AB)^{1/2}$

14. **(c) :** $\vec{A}\times 0$ is a zero vector.

15. **(b) :** $\vec{v} = \vec{\omega}\times\vec{r} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 5 & -6 & 6\\ 3 & -4 & 1\end{vmatrix} = 18\hat{i}+13\hat{j}-2\hat{k}$

16. **(c) :** Here, $\vec{F} = 3\hat{j}$ N, $\vec{r} = 2\hat{k}$ m

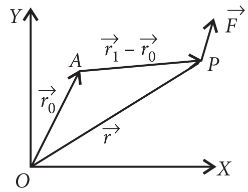
Torque, $\vec{\tau} = \vec{r}\times\vec{F} = 2\hat{k}\times 3\hat{j} = -6\hat{i}$ N m

17. **(d) :** Moment of the force is,

$\vec{\tau} = (\vec{r}-\vec{r}_0)\times\vec{F}$

Here, $\vec{r}_0 = 2\hat{i}-2\hat{j}-2\hat{k}$

and $\vec{r} = 2\hat{i}+0\hat{j}-3\hat{k}$



$\therefore \quad \vec{r}-\vec{r}_0 = (2\hat{i}+0\hat{j}-3\hat{k})-(2\hat{i}-2\hat{j}-2\hat{k}) = 0\hat{i}+2\hat{j}-\hat{k}$

$\therefore \quad \vec{\tau} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 2 & -1\\ 4 & 5 & -6\end{vmatrix} = -7\hat{i}-4\hat{j}-8\hat{k}$

18. **(c) :** For the conservation of angular momentum about origin, the torque $\vec{\tau}$ acting on the particle will be zero.

By definition, $\vec{\tau} = \vec{r}\times\vec{F}$

Here, $\vec{r} = 2\hat{i}-6\hat{j}-12\hat{k}$ and $\vec{F} = \alpha\hat{i}+3\hat{j}+6\hat{k}$

$\therefore \quad \vec{\tau} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -6 & -12\\ \alpha & 3 & 6\end{vmatrix}$

$= \hat{i}(-36+36)-\hat{j}(12+12\alpha)+\hat{k}(6+6\alpha)$

$= -\hat{j}(12+12\alpha)+\hat{k}(6+6\alpha)$

But $\vec{\tau} = 0$

$\therefore \quad 12+12\alpha = 0 \quad \text{or} \quad \alpha = -1$

and $6+6\alpha = 0 \quad \text{or} \quad \alpha = -1$

19. **(a) :** According to law of conservation of angular momentum

$mvr = mv'r'$

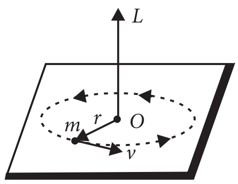
$v_0R_0 = v\left(\dfrac{R_0}{2}\right)$; $v = 2v_0$ ...(i)

$$\therefore \quad \frac{K_0}{K} = \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}mv^2} = \left(\frac{v_0}{v}\right)^2$$

or $\dfrac{K}{K_0} = \left(\dfrac{v}{v_0}\right)^2 = (2)^2$ (Using (i))

$K = 4K_0 = 2mv_0^{\ 2}$

20. (a) : When a mass is rotating in a plane about a fixed point its angular momentum is directed along a line perpendicular to the plane of rotation.

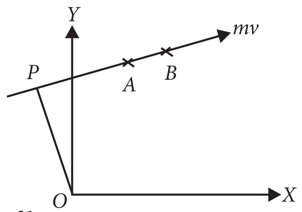


21. (d)

22. (b) : Torque is always perpendicular to $\vec{F}$ as well as $\vec{r}$.

$\vec{r}\cdot\vec{\tau} = 0$ as well as $\vec{F}\cdot\vec{\tau} = 0$.

23. (a) :



Moment of linear momentum is angular momentum. OP is the same whether the mass is at A or B.

$\therefore \quad L_A = L_B$.

24. (c) : Force $(\vec{F}) = -3\hat{i} + \hat{j} + 5\hat{k}$ and distance of the point $(\vec{r}) = 7\hat{i} + 3\hat{j} + \hat{k}$

$$\text{Torque } \vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

25. (d) : Force $(\vec{F}) = 2\hat{i} - 3\hat{j} + 4\hat{k}$ N and distance of the point from origin $(\vec{r}) = 3\hat{i} + 2\hat{j} + 3\hat{k}$ m.

Torque $\vec{\tau} = \vec{r}\times\vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 3 \\ 2 & -3 & 4 \end{vmatrix} = 17\hat{i} - 6\hat{j} - 13\hat{k}$$

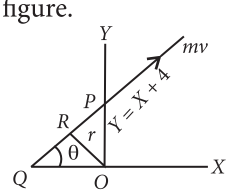
26. (a) : $\vec{L} = \vec{r}\times\vec{p}$

$y = x + 4$ line has been shown in the figure.

When $x = 0$,

$y = 4$, So $OP = 4$.

The slope of the line can be obtained by comparing with the equation of line



$y = mx + c$

$m = \tan\theta = 1 \quad \Rightarrow \quad \theta = 45°$

$\angle OQP = \angle OPQ = 45°$

If we draw a line perpendicular to this line.

Length of the perpendicular $= OR$

$$\Rightarrow \quad OR = OP\sin 45° = 4\frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Angular momentum of particle going along this line

$= r\times mv = 2\sqrt{2}\times 5\times 3\sqrt{2} = 60$ units

27. (*) : Centre of gravity of a body is the point at which the total gravitational torque on body is zero. Centre of mass and centre of gravity coincides only for symmetrical bodies.

Hence statements (1) and (2) are incorrect.

A couple of a body produces rotational motion only. Hence statement (3) is incorrect.

Mechanical advantage is greater than one means that the system will require a force that is less than the load in order to move it.

Hence statement (4) is correct.

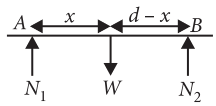
*None of the given options is correct.

28. (b) : Given situation is shown in figure.

N_1 = Normal reaction on A

N_2 = Normal reaction on B

W = Weight of the rod



In vertical equilibrium,

$N_1 + N_2 = W$ ...(i)

Torque balance about centre of mass of the rod,

$N_1 x = N_2(d - x)$

Putting value of N_2 from equation (i)

$N_1 x = (W - N_1)(d - x) \Rightarrow N_1 x = Wd - Wx - N_1 d + N_1 x$

$$\Rightarrow \quad N_1 d = W(d - x); \ \therefore \ N_1 = \frac{W(d - x)}{d}$$

29. (a) : Let x be the perpendicular distance of centre O of equilateral triangle from each side.

Total torque about $O = 0$

$\Rightarrow \quad F_1 x + F_2 x - F_3 x = 0 \quad$ or $\quad F_3 = F_1 + F_2$

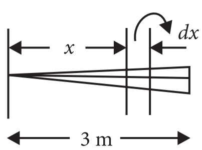
30. (a)

31. (b) : Let us consider an elementary length dx at a distance x from one end.

Its mass $= k\cdot x\cdot dx$

[k = proportionality constant]

Then centre of gravity of the rod x_c is given by



$$x_c = \frac{\int_0^3 kxdx\cdot x}{\int_0^3 kxdx} = \frac{\int_0^3 x^2dx}{\int_0^3 xdx} = \frac{\left.\frac{x^3}{3}\right|_0^3}{\left.\frac{x^2}{2}\right|_0^3}$$

or $x_c = \dfrac{27/3}{9/2} = 2.$

$\therefore$ Centre of gravity of the rod will be at distance of 2 m from one end.

32. (c) : Load $W = Mg = 75 \times 10 = 750$ N

Effort $P = 250$ N

$\therefore$ Mechanical advantage

$$= \frac{\text{load}}{\text{effort}} = \frac{W}{P} = \frac{750}{250} = 3$$

Velocity ratio

$$= \frac{\text{distance travelled by effort}}{\text{distance travelled by load}} = \frac{12}{3} = 4$$

$$\text{Efficiency, } \eta = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$

$$= (3/4) \times 100 = 75\%.$$

33. (d)

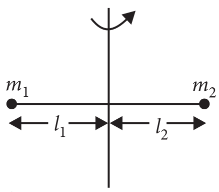
34. (b) : $\frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}} = \frac{\frac{1}{2} I_s \omega_s^2}{\frac{1}{2} I_c \omega_c^2} = \frac{I_s \omega_s^2}{I_c \omega_c^2}$

Here, $I_s = \frac{2}{5} MR^2, \ I_c = \frac{1}{2} MR^2, \omega_c = 2\omega_s$

$$\frac{E_{\text{Sphere}}}{E_{\text{Cylinder}}} = \frac{\frac{2}{5} mR^2 \times \omega_s^2}{\frac{1}{2} mR^2 \times (2\omega_s)^2} = \frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$$

35. (a) : Here, $l_1 + l_2 = l$

Centre of mass of the system,



$$l_1 = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2} = \frac{m_2 l}{m_1 + m_2}$$

$$l_2 = l - l_1 = \frac{m_1 l}{m_1 + m_2}$$

Required moment of inertia of the system,

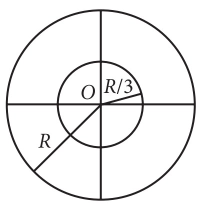
$$I = m_1 l_1^2 + m_2 l_2^2$$

$$= (m_1 m_2^2 + m_2 m_1^2) \frac{l^2}{(m_1 + m_2)^2}$$

$$= \frac{m_1 m_2 (m_1 + m_2) l^2}{(m_1 + m_2)^2} = \frac{m_1 m_2}{m_1 + m_2} l^2$$

36. (a) : Mass of the disc = $9M$

Mass of removed portion of disc = M



The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is $I_1 = \frac{9}{2} MR^2$

Now, the moment of inertia of the removed portion of the disc

$$I_2 = \frac{1}{2} M \left(\frac{R}{3}\right)^2 = \frac{1}{18} MR^2$$

Therefore, moment of inertia of the remaining portion of disc about O is

$$I = I_1 - I_2 = 9\frac{MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$$

37. (d) : M.I. of a circular disc, $Mk^2 = \frac{M \cdot R^2}{2}$

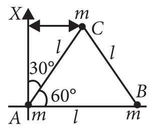
M.I. of a circular ring = MR^2.

$\therefore$ Ratio of their radius of gyration = $\frac{1}{\sqrt{2}} : 1 = 1 : \sqrt{2}$

38. (c) : K.E. $= \frac{1}{2} I\omega^2$

$$\therefore \ \frac{1}{2} I\omega_1^2 = \frac{1}{2} \cdot 2I\omega_2^2; \ \frac{\omega_1^2}{\omega_2^2} = \frac{2}{1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{\sqrt{2}}{1}.$$

39. (c) : The moment of inertia of the system



$= m_A r_A^2 + m_B r_B^2 + m_C r_C^2$

$= m_A(0)^2 + m(l)^2 + m(l\sin 30°)^2$

$= ml^2 + ml^2 \times (1/4) = (5/4)\, ml^2$

40. (a) : A circular disc may be divided into a large number of circular rings. Moment of inertia of the disc will be the summation of the moments of inertia of these rings about the geometrical axis. Now, moment of inertia of a circular ring about its geometrical axis is MR^2, where M is the mass and R is the radius of the ring.

Since the density (mass per unit volume) for iron is more than that of aluminium, the proposed rings made of iron should be placed at a higher radius to get more value of MR^2. Hence to get maximum moment of inertia for the circular disc, aluminium should be placed at interior and iron at the exterior position.

41. (b) : As effective distance of mass from BC is greater than the effective distance of mass from AB, therefore $I_2 > I_1$.

42. (d) : The intersection of medians is the centre of mass of the triangle. Since the distances of centre of mass from the sides is related as $x_{BC} < x_{AB} < x_{AC}$.

Therefore $I_{BC} > I_{AB} > I_{AC}$ or $I_{BC} > I_{AB}$.

43. (d) : The moment of inertia is minimum about EG because mass distribution is at minimum distance from EG.

44. (c) : K.E. $= \frac{1}{2} I\omega^2$

$$I = \frac{2\,\text{K.E.}}{\omega^2} = \frac{2 \times 360}{30 \times 30} = 0.8 \text{ kg m}^2$$

45. (a) : Kinetic energy = $\frac{1}{2} I\omega^2$, and for ring $I = mr^2$.

Hence $\text{KE} = \frac{1}{2} mr^2\omega^2$

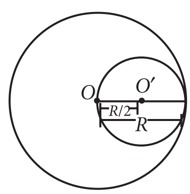
46. (d) : Mass per unit area of disc = $\frac{M}{\pi R^2}$

Mass of removed portion of disc,

$$M' = \frac{M}{\pi R^2} \times \pi \left(\frac{R}{2}\right)^2 = \frac{M}{4}$$

Moment of inertia of removed portion about an axis passing through centre of disc O and perpendicular to the plane of disc,

$$I'_O = I_{O'} + M'd^2$$



$$= \frac{1}{2} \times \frac{M}{4} \times \left(\frac{R}{2}\right)^2 + \frac{M}{4} \times \left(\frac{R}{2}\right)^2$$

$$= \frac{MR^2}{32} + \frac{MR^2}{16} = \frac{3MR^2}{32}$$

The moment of inertia of complete disc about centre O before removing the portion of the disc

$$I_O = \frac{1}{2}MR^2$$

So, moment of inertia of the disc with removed portion is

$$I = I_O - I'_O = \frac{1}{2}MR^2 - \frac{3MR^2}{32} = \frac{13MR^2}{32}$$

47. (b) : Net moment of inertia of the system,

$$I = I_1 + I_2 + I_3.$$

The moment of inertia of a shell about its diameter,

$$I_1 = \frac{2}{3}mr^2.$$

The moment of inertia of a shell about its tangent is given by

$$I_2 = I_3 = I_1 + mr^2 = \frac{2}{3}mr^2 + mr^2 = \frac{5}{3}mr^2$$

$$\therefore \quad I = 2\times\frac{5}{3}mr^2 + \frac{2}{3}mr^2 = \frac{12mr^2}{3} = 4mr^2$$

48. (a) : According to the theorem of parallel axes, $I = I_{CM} + Ma^2$.

As a is maximum for point B. Therefore I is maximum about B.

49. (b) : According to the theorem of parallel axes, the moment of inertia of the thin rod of mass M and length L about an axis passing through one of the ends is

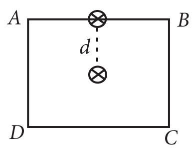
$$I = I_{CM} + Md^2$$

where I_{CM} is the moment of inertia of the given rod about an axis passing through its centre of mass and perpendicular to its length and d is the distance between two parallel axes.

Here, $I_{CM} = I_0,\ d = \frac{L}{2}$

$$\therefore \quad I = I_0 + M\left(\frac{L}{2}\right)^2 = I_0 + \frac{ML^2}{4}$$

50. (d) : Moment of inertia for the rod AB rotating about an axis through the mid-point of AB perpendicular to the plane of the paper is $\frac{Ml^2}{12}$.

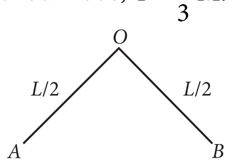


$\therefore$ Moment of inertia about the axis through the centre of the square and parallel to this axis,

$$I = I_0 + Md^2 = M\left(\frac{l^2}{12} + \frac{l^2}{4}\right) = \frac{Ml^2}{3}.$$

For all the four rods, $I = \frac{4}{3}Ml^2$

51. (d) :



Total mass = M, total length = L

Moment of inertia of OA about O = Moment of inertia of OB about O

$$\Rightarrow \text{M.I.}_{\text{total}} = 2\times\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2\cdot\frac{1}{3} = \frac{ML^2}{12}$$

52. (d) : Moment of inertia of a uniform circular disc about an axis through its centre and perpendicular to its plane is $I_C = \frac{1}{2}MR^2$

$\therefore$ Moment of inertia of a uniform circular disc about an axis touching the disc at its diameter and normal to the disc is I.

By the theorem of parallel axes,

$$I = I_C + Mh^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

53. (c) : Radius of gyration of disc about a tangential axis in the plane of disc is $\frac{\sqrt{5}}{2}R = K_1$, radius of gyration of circular ring of same radius about a tangential axis in the plane of circular ring is

$$K_2 = \sqrt{\frac{3}{2}}R \quad \therefore \quad \frac{K_1}{K_2} = \frac{\sqrt{5}}{\sqrt{6}}.$$

54. (a) : Moment of inertia of a disc about its diameter $= \frac{1}{4}MR^2$

Using theorem of parallel axes,

$$I = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

55. (c) : Moment of inertia of uniform circular disc about diameter = I

According to theorem of perpendicular axes.

Moment of inertia of disc about axis

$$= 2I = \frac{1}{2}mr^2$$

Using theorem of parallel axes,

Moment of inertia of disc about the given axis

$$= 2I + mr^2 = 2I + 4I = 6I$$

56. (a) : Given: Angular acceleration, $\alpha = 3$ rad/s^2

Initial angular velocity $\omega_i = 2$ rad/s

Time $t = 2$ s

Using, $\theta = \omega_i t + \frac{1}{2}\alpha t^2$

$$\therefore \quad \theta = 2\times2 + \frac{1}{2}\times3\times4 = 4+6 = 10 \text{ radians}$$

57. (b) : Given : Mass $M = 2$ kg, Radius $R = 4$ cm

Initial angular speed

$$\omega_0 = 3\,\text{rpm} = 3\times\frac{2\pi}{60}\,\text{rad/s} = \frac{\pi}{10}\ \text{rad/s}$$

We know that, $\omega^2 = \omega_0^2 + 2\alpha\theta$

$$\Rightarrow \quad 0 = \left(\frac{\pi}{10}\right)^2 + 2\times\alpha\times2\pi\times2\pi \quad \Rightarrow \quad \alpha = \frac{-1}{800}\,\text{rad/s}^2$$

Moment of inertia of a solid cylinder,

$$I = \frac{MR^2}{2} = \frac{2\times\left(\frac{4}{100}\right)^2}{2} = \frac{16}{10^4}$$

Torque $\tau = I\alpha = \left(\frac{16}{10^4}\right)\times\left(-\frac{1}{800}\right) = -2\times10^{-6}$ N m

58. (a) : Work done required to bring a object to rest
$\Delta W = \Delta KE$

$\Delta W = \frac{1}{2}I\omega^2$; where I = moment of inertia

For same ω, $\Delta W \propto I$

For a solid sphere, $I_A = \frac{2}{5}MR^2$

For a thin circular disk, $I_B = \frac{1}{2}MR^2$

For a circular ring, $I_C = MR^2$

$\therefore \quad I_C > I_B > I_A \quad \therefore \quad W_C > W_B > W_A$

59. (b) : Here, m = 3 kg, r = 40 cm = 40 × 10^{-2} m, F = 30 N

Moment of inertia of hollow cylinder about its axis
$= mr^2 = 3 \text{ kg} \times (0.4)^2 \text{ m}^2 = 0.48 \text{ kg m}^2$

The torque is given by, $\tau = I\alpha$

where I = moment of inertia, α = angular acceleration

In the given case, $\tau = rF$, as the force is acting perpendicularly to the radial vector.

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3\times40\times10^{-2}} = \frac{30\times100}{3\times40}$

$\alpha = 25 \text{ rad s}^{-2}$

60. (c) : Given, r = 50 cm = 0.5 m, α = 2.0 rad s^{-2}, $\omega_0 = 0$

At the end of 2 s,

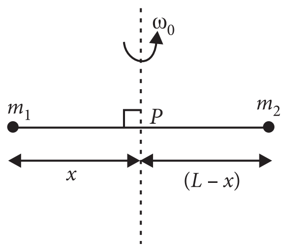
tangential acceleration, $a_t = r\alpha = 0.5 \times 2 = 1 \text{ m s}^{-2}$

radial acceleration, $a_r = \omega^2 r = (\omega_0 + \alpha t)^2 r$
$= (0 + 2\times2)^2 \times 0.5 = 8 \text{ m s}^{-2}$

$\therefore$ Net acceleration,

$a = \sqrt{a_t^2 + a_r^2} = \sqrt{1^2 + 8^2} = \sqrt{65} \approx 8 \text{ m s}^{-2}$

61. (b) :



Moment of inertia of the system about the axis of rotation (through point P) is

$I = m_1x^2 + m_2(L-x)^2$

By work energy theorem,

Work done to set the rod rotating with angular velocity ω_0 = Increase in rotational kinetic energy

$W = \frac{1}{2}I\omega_0^2 = \frac{1}{2}[m_1x^2 + m_2(L-x)^2]\omega_0^2$

For W to be minimum, $\frac{dW}{dx} = 0$

i.e. $\frac{1}{2}[2m_1x + 2m_2(L-x)(-1)]\omega_0^2 = 0$

or $\quad m_1x - m_2(L-x) = 0 \qquad (\because \omega_0 \neq 0)$

or $\quad (m_1 + m_2)x = m_2L$ or $x = \frac{m_2L}{m_1+m_2}$

62. (c) : Here, speed of the automobile,

$v = 54 \text{ km h}^{-1} = 54\times\frac{5}{18} \text{ m s}^{-1} = 15 \text{ m s}^{-1}$

Radius of the wheel of the automobile, R = 0.45 m

Moment of inertia of the wheel about its axis of rotation, $I = 3 \text{ kg m}^2$

Time in which the vehicle brought to rest, t = 15 s

The initial angular speed of the wheel is

$\omega_i = \frac{v}{R} = \frac{15 \text{ m s}^{-1}}{0.45 \text{ m}} = \frac{1500}{45}\text{rad s}^{-1} = \frac{100}{3} \text{ rad s}^{-1}$

and its final angular speed is

$\omega_f = 0$ (as the vehicle comes to rest)

$\therefore$ The angular retardation of the wheel is

$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - \frac{100}{3}}{15 \text{ s}} = -\frac{100}{45} \text{ rad s}^{-2}$

The magnitude of required torque is

$\tau = I|\alpha| = (3 \text{ kg m}^2)\left(\frac{100}{45}\text{rad s}^{-2}\right)$

$= \frac{20}{3} \text{ kg m}^2\text{s}^{-2} = 6.66 \text{ kg m}^2\text{s}^{-2}$

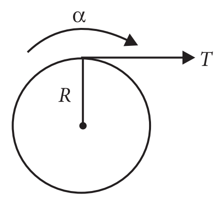
63. (d) : Here, mass of thecylinder, M = 50 kg

Radius of the cylinder, R = 0.5 m

Angular acceleration, α = 2 rev s^{-2}
$= 2 \times 2\pi$ rad s^{-2} = 4π rad s^{-2}

Torque, $\tau = TR$

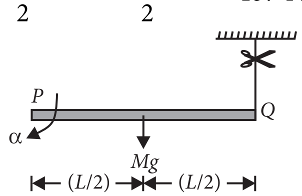
Moment of inertia of the solid cylinder about its axis, $I = \frac{1}{2}MR^2$



$\therefore$ Angular acceleration of the cylinder, $\alpha = \frac{\tau}{I} = \frac{TR}{\frac{1}{2}MR^2}$

$T = \frac{MR\alpha}{2} = \frac{50\times0.5\times4\pi}{2} = 157$ N

64. (c) :



When the string is cut, the rod will rotate about P. Let α be initial angular acceleration of the rod. Then

Torque, $\tau = I\alpha = \frac{ML^2}{3}\alpha$ ...(i)

Also, $\tau = Mg\frac{L}{2}$ ...(ii)

Equating (i) and (ii), we get

$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha$ or $\alpha = \frac{3g}{2L}$

65. (a) : Given : $\theta(t) = 2t^3 - 6t^2$

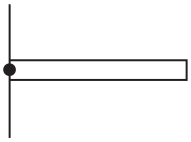
$\therefore \quad \frac{d\theta}{dt} = 6t^2 - 12t \Rightarrow \frac{d^2\theta}{dt^2} = 12t - 12$

Angular acceleration, $\alpha = \frac{d^2\theta}{dt^2} = 12t - 12$

When angular acceleration (α) is zero, then the torque on the wheel becomes zero $(\because \tau = I\alpha)$

$\Rightarrow$ $12t - 12 = 0$ or $t = 1$ s

66. (c) : Torque about A,



$$\tau = mg \times \frac{l}{2} = \frac{mgl}{2}$$

Also $\tau = I\alpha$

$\therefore$ Angular acceleration,

$$\alpha = \frac{\tau}{I} = \frac{mgl/2}{ml^2/3} = \frac{3}{2}\frac{g}{l}.$$

67. (c) : $\omega_f = \omega_i - \alpha t \Rightarrow 0 = \omega_i - \alpha_t$

$\therefore$ $\alpha = \omega_i/t$, where α is retardation.

The torque on the wheel is given by

$$\tau = I\alpha = \frac{I\omega}{t} = \frac{I \cdot 2\pi\upsilon}{t} = \frac{2 \times 2 \times \pi \times 60}{60 \times 60} = \frac{\pi}{15} \text{ N m}$$

68. (b) : $I = 1.2$ kg m^2, $E_r = 1500$ J,

$\alpha = 25$ rad/s^2, $\omega_1 = 0$, $t = ?$

As $E_r = \frac{1}{2}I\omega^2$, $\omega = \sqrt{\frac{2E_r}{I}} = \sqrt{\frac{2 \times 1500}{1.2}} = 50$ rad/s

From $\omega_2 = \omega_1 + \alpha t$

$50 = 0 + 25t$, or $t = 2$ s

69. (d) : As there is no external torque acting on a sphere, *i.e.*, $\tau_{ex} = 0$

So, $\frac{dL}{dt} = \tau_{ex} = 0$ *i.e.*, L = constant

So angular momentum remains constant.

70. (a) : Initial angular momentum = $I\omega_1 + I\omega_2$

Let ω be angular speed of the combined system.

Final angular momentum = $2I\omega$

$\therefore$ According to conservation of angular momentum

$$I\omega_1 + I\omega_2 = 2I\omega \quad \text{or} \quad \omega = \frac{\omega_1 + \omega_2}{2}$$

Initial rotational kinetic energy,

$$E = \frac{1}{2}I(\omega_1^2 + \omega_2^2)$$

Final rotational kinetic energy

$$E_f = \frac{1}{2}(2I)\omega^2 = \frac{1}{2}(2I)\left(\frac{\omega_1 + \omega_2}{2}\right)^2 = \frac{1}{4}I(\omega_1 + \omega_2)^2$$

$\therefore$ Loss of energy $\Delta E = E_i - E_f$

$$= \frac{I}{2}(\omega_1^2 + \omega_2^2) - \frac{I}{4}(\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2)$$

$$= \frac{I}{4}\left[\omega_1^2 + \omega_2^2 - 2\omega_1\omega_2\right] = \frac{I}{4}(\omega_1 - \omega_2)^2$$

71. (c) : Here, $m_A = m$, $m_B = 2m$

Both bodies A and B have equal kinetic energy of rotation

$$K_A = K_B \Rightarrow \frac{1}{2}I_A\omega^2{}_A = \frac{1}{2}I_B\omega^2{}_B$$

$$\Rightarrow \frac{\omega_A^2}{\omega_B^2} = \frac{I_B}{I_A} \quad \text{...(i)}$$

Ratio of angular momenta,

$$\frac{L_A}{L_B} = \frac{I_A\omega_A}{I_B\omega_B} = \frac{I_A}{I_B} \times \sqrt{\frac{I_B}{I_A}} \quad \text{[Using eqn. (i)]}$$

$$= \sqrt{\frac{I_A}{I_B}} < 1 \quad (\because I_B > I_A)$$

$\therefore$ $L_B > L_A$

72. (b) : Moment of inertia of disc D_1 about an axis passing through its centre and normal to its plane is

$$I_1 = \frac{MR^2}{2} = \frac{(2\text{ kg})(0.2\text{ m})^2}{2} = 0.04\text{ kg m}^2$$

Initial angular velocity of disc D_1, $\omega_1 = 50$ rad s^{-1}

Moment of inertia of disc D_2 about an axis passing through its centre and normal to its plane is

$$I_2 = \frac{(4\text{ kg})(0.1\text{ m})^2}{2} = 0.02\text{ kg m}^2$$

Initial angular velocity of disc D_2, $\omega_2 = 200$ rad s^{-1}

Total initial angular momentum of the two discs is

$$L_i = I_1\omega_1 + I_2\omega_2$$

When two discs are brought in contact face to face (one on the top of the other) and their axes of rotation coincide, the moment of inertia I of the system is equal to the sum of their individual moment of inertia.

$$I = I_1 + I_2$$

Let ω be the final angular speed of the system. The final angular momentum of the system is

$$L_f = I\omega = (I_1 + I_2)\omega$$

According to law of conservation of angular momentum, we get

$$L_i = L_f \text{ or, } I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega \text{ or, } \omega = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$= \frac{(0.04\text{ kg m}^2)(50\text{ rad s}^{-1}) + (0.02\text{ kg m}^2)(200\text{ rad s}^{-1})}{(0.04 + 0.02)\text{ kg m}^2}$$

$$= \frac{(2+4)}{0.06}\text{ rad s}^{-1} = 100\text{ rad s}^{-1}$$

73. (c) : As the system is initially at rest, therefore, initial angular momentum $L_i = 0$.

According to the principle of conservation of angular momentum, final angular momentum, $L_f = 0$.

$\therefore$ Angular momentum of platform = Angular momentum of man in opposite direction of platform.

i.e., $mvR = I\omega$

$$\text{or} \quad \omega = \frac{mvR}{I} = \frac{50 \times 1 \times 2}{200} = \frac{1}{2}\text{ rad s}^{-1}$$

Angular velocity of man relative to platform is

$$\omega_r = \omega + \frac{v}{R} = \frac{1}{2} + \frac{1}{2} = 1\text{ rad s}^{-1}$$

Time taken by the man to complete one revolution is

$$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{1} = 2\pi \text{ s}$$

74. (d) : As no external torque is applied to the system, the angular momentum of the system remains conserved.

$\therefore \quad L_i = L_f$

According to given problem,

$$I_t\omega_i = (I_t + I_b)\omega_f \quad \text{or} \quad \omega_f = \frac{I_t\omega_i}{(I_t + I_b)} \quad \text{...(i)}$$

Initial energy, $E_i = \frac{1}{2}I_t\omega_i^2$ ...(ii)

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\omega_f^2$ ...(iii)

Substituting the value of ω_f from equation (i) in equation (iii), we get

Final energy,

$$E_f = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\omega_i}{I_t + I_b}\right)^2 = \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)} \quad \text{...(iv)}$$

Loss of energy, $\Delta E = E_i - E_f$

$$= \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)} \quad \text{(Using (ii) and (iv))}$$

$$= \frac{\omega_i^2}{2}\left(I_t - \frac{I_t^2}{(I_t + I_b)}\right) = \frac{\omega_i^2}{2}\left(\frac{I_t^2 + I_bI_t - I_t^2}{(I_t + I_b)}\right)$$

$$= \frac{1}{2}\frac{I_bI_t}{(I_t + I_b)}\omega_i^2$$

75. (d) : As no external torque is acting about the axis, angular momentum of system remains conserved.

$$I_1\omega_1 = I_2\omega_2$$

$$\Rightarrow \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mr^2\omega}{(M + 2m)r^2} = \frac{M\omega}{(M + 2m)}$$

76. (c) : Applying conservation of angular momentum

$$I_1\omega = (I_1 + I_2)\omega_1 \quad \text{or} \quad \omega_1 = \frac{I_1}{(I_1 + I_2)}\omega$$

77. (b) : According to conservation of angular momentum, $L = I\omega$ = constant

Therefore, $I_2\omega_2 = I_1\omega_1$

$$\text{or} \quad \omega_2 = \frac{I_1\omega_1}{I_2} = \frac{Mr^2\omega}{(M + 4m)r^2} = \frac{M\omega}{M + 4m}.$$

78. (b) : When a child sits on a rotating disc, no external torque is introduced. Hence the angular momentum of the system is conserved. But the moment of inertia of the system will increase and as a result, the angular speed of the disc will decrease to maintain constant angular momentum.

79. (b) : Required work done = – $(K_f - K_i) = 0 + K_i = K_i$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\frac{v^2}{R^2} + \frac{1}{2}mv^2$$

$$= \frac{3}{4}mv^2 = \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2 = 3 \text{ J}$$

80. (d) : Using law of conservation of energy,

$$mgh = \frac{1}{2}mv_{cm}^2 + \frac{1}{4}mr^2\omega^2$$

$$\Rightarrow mg\, s \sin 30^\circ = \frac{1}{2}mv_{cm}^2 + \frac{1}{4}mv_{cm}^2$$

$$\Rightarrow s = \frac{3}{4}\frac{v_{cm}^2}{g \sin 30^\circ} \quad \Rightarrow \quad s = \frac{3}{4} \times \frac{4^2}{10 \times \frac{1}{2}}$$

$$\Rightarrow s = \frac{3 \times 4 \times 2}{10} = \frac{12}{5} = 2.4 \text{ m}$$

81. (b) : Translational kinetic energy, $K_t = \frac{1}{2}mv^2$

Rotational kinetic energy, $K_r = \frac{1}{2}I\omega^2$

$$\therefore K_t + K_r = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$\left[\because I = \frac{2}{5}mr^2 \text{ (for sphere)}\right]$$

$$\therefore \quad K_t + K_r = \frac{7}{10}mv^2$$

So, $\frac{K_t}{K_t + K_r} = \frac{5}{7}$

82. (d) : Time taken by the body to reach the bottom when it rolls down on an inclined plane without slipping is given by

$$t = \sqrt{\frac{2l\left(1 + \frac{k^2}{R^2}\right)}{g \sin\theta}}$$

Since g is constant and l, R and $\sin\theta$ are same for both

$$\therefore \frac{t_d}{t_s} = \frac{\sqrt{1 + \frac{k_d^2}{R^2}}}{\sqrt{1 + \frac{k_s^2}{R^2}}} = \sqrt{\frac{1 + \frac{R^2}{2R^2}}{1 + \frac{2R^2}{5R^2}}} \quad \left(\because k_d = \frac{R}{\sqrt{2}}, k_s = \sqrt{\frac{2}{5}}R\right)$$

$$= \sqrt{\frac{3}{2} \times \frac{5}{7}} = \sqrt{\frac{15}{14}} \Rightarrow t_d > t_s$$

Hence, the sphere gets to the bottom first.

83. (a) : Acceleration of the solid sphere slipping down the incline without rolling is

$$a_{\text{slipping}} = g\sin\theta \quad \text{...(i)}$$

Acceleration of the solid sphere rolling down the incline without slipping is

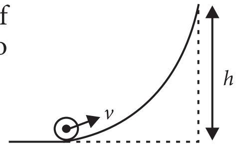
$$a_{\text{rolling}} = \frac{g\sin\theta}{1 + \frac{k^2}{R^2}} = \frac{g\sin\theta}{1 + \frac{2}{5}} = \frac{5}{7}g\sin\theta \quad \text{...(ii)}$$

$$\left(\because \text{For solid sphere, } \frac{k^2}{R^2} = \frac{2}{5}\right)$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{a_{\text{rolling}}}{a_{\text{slipping}}} = \frac{5}{7}$$

84. (b) : The kinetic energy of the rolling object is converted into potential energy at height h and



$$h = \frac{3v^2}{4g}$$

So by the law of conservation of mechanical energy, we have

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = Mg\left(\frac{3v^2}{4g}\right)$$

$$\frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}Mv^2 - \frac{1}{2}Mv^2$$

$$\frac{1}{2}I\frac{v^2}{R^2} = \frac{1}{4}Mv^2 \quad \text{or} \quad I = \frac{1}{2}MR^2$$

Hence, the object is disc.

85. (b) : At maximum compression the solid cylinder will stop.

According to law of conservation of mechanical energy

Loss in kinetic energy of cylinder = Gain in potential of spring

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2;\ \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{mR^2}{2}\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}kx^2$$

$(\because\ v = R\omega$ and for solid cylinder, $I = \frac{1}{2}mR^2)$

$$\frac{1}{2}mv^2 + \frac{1}{4}mv^2 = \frac{1}{2}kx^2 \text{ or, } \frac{3}{4}mv^2 = \frac{1}{2}kx^2 \text{ or } x^2 = \frac{3}{2}\frac{mv^2}{k}$$

Here, $m = 3$ kg, $v = 4$ m s^{-1}, $k = 200$ N m^{-1}

Substituting the given values, we get

$$x^2 = \frac{3\times3\times4\times4}{2\times200} \Rightarrow x^2 = \frac{36}{100} \text{ or } x = 0.6 \text{ m}$$

86. (d) : Time taken to reach the bottom of inclined plane.

$$t = \sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

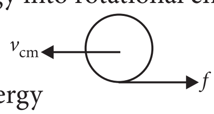
Here, l is length of incline plane.

For solid cylinder $k^2 = \frac{R^2}{2}$.

For hollow cylinder $k^2 = R^2$.

Hence, solid cylinder will reach the bottom first.

87. (d) : Required frictional force convert some part of translational energy into rotational energy.



88. (c) : Total energy

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv^2(1 + K^2/R^2)$$

Required fraction $= \dfrac{K^2/R^2}{1+K^2/R^2} = \dfrac{K^2}{R^2+K^2}$

89. (c) : Potential energy of the solid cylinder at height $h = Mgh$

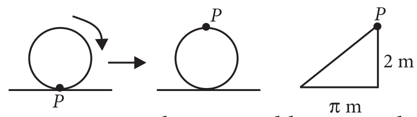
K.E. of centre of mass when it reaches the bottom

$$= \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\frac{v^2}{R^2} = \frac{1}{2}Mv^2\left(1+\frac{k^2}{R^2}\right)$$

For a solid cylinder $\frac{k^2}{R^2} = \frac{1}{2}$ $\quad\therefore$ K.E. $= \frac{3}{4}Mv^2$

$$\therefore\ Mgh = \frac{3}{4}Mv^2,\ v = \sqrt{\frac{4}{3}gh}$$

90. (b) :



In half rotation point P has moved horizontal distance

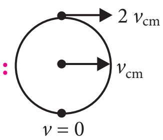
$$\frac{\pi d}{2} = \pi r = \pi\times1 \text{ m} = \pi \text{ m} \qquad [\because \text{ radius} = 1 \text{ m}]$$

In the same time, it has moved vertical distance which is equal to its diameter = 2 m

$\therefore$ Displacement of point $P = \sqrt{\pi^2+2^2} = \sqrt{\pi^2+4}$ m

91. (d) : Since there is no friction at the contact surface (smooth horizontal surface) there will be no rolling. Hence, the acceleration of the centre of mass of the sphere will be independent of the position of the applied force F. Therefore, there is no relation between h and R.

92. (a) :



93. (d) : Linear K.E. of ball $= \frac{1}{2}mv^2$ and

rotational K.E. of ball $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 = \frac{1}{5}mv^2$

Total K.E. $= \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$

Ratio of rotational K.E. and total K.E.

$$= \frac{(1/5)mv^2}{(7/10)mv^2} = \frac{2}{7}$$

94. (a) : For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

For disc and solid cylinder, $\frac{K^2}{R^2} = \frac{1}{2}$

As for solid sphere K^2/R^2 is smallest, it takes minimum time to reach the bottom of the incline, disc and cylinder reach together later.

95. (a) : P.E. = total K.E.

$$mgh = \frac{7}{10}mv^2,\ v = \sqrt{\frac{10gh}{7}}$$

96. (b) : Angular momentum about the point of contact with the surface includes the angular momentum about the centre. Because of friction, linear momentum will not be conserved.

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